Approaching Completeness

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In an essay titled "I Want to Find The Music, Not to Compose It", Tom Johnson writes about a type of musical minimalism he remembers as "process music", saying that "in [all] these cases the "composers" are not really composing so much as simply letting music arise out of circumstances that they can not personally control. They are finding music which somehow already exists."[i] Johnson is referring to the music of composers in 1970s downtown NYC, but he opens up the discussion to reflect on an approach of "found" music that can be applied to what one could consider a "musical catalogue". Taking this idea of "found music" as our starting point, we can offer a sufficient definition of what we mean by the term "musical catalogue" that, while necessarily incomplete, can serve as our guide:

A musical catalogue is a musical work that contains all of the instances or possibilities of a compositional design, progression, melody, harmony, rhythm, or other parametrically constrained musical object.

The Chord Catalogue, composed in 1985 by Johnson, is the most prominent example of the last thirty years from the world of pure, systematic music. Johnson wrote about this piece:

The Chord Catalogue consists of the 8178 chords possible in one octave. It is really just a list. The chords are simply stated, in a logical sequence, rather than being composed, and the main concern of the piece is to remain open to all sounds, all harmonies. It is fine to have personal preferences, and to feel that some sounds are more beautiful than other sounds, but it is also good to realize that there are an enormous number of possible chords, and that each one has something just a little special about it.[ii]

In *The Chord Catalogue*, Johnson approaches his "list" of chords in a straightforward, logical manner: he organizes the 8178 chords into groupings of two-note chords, three-note chords, four-note chords, and so on, until he reaches the one thirteen-note chord that completes the octave. Within each grouping, the chords are presented in order so that there is no duplication within the collection of n-note chords. Thus, in the first phrase of the two-note chords, we first hear notes (1,2) together, then, in the second phrase, (1,3) and (2,3), and so forth. This excerpt goes to the sixth phrase:

This didactic approach allows for the list to simply be heard unadorned by any other parametric constraints like dynamics, articulations, and embellishments, yet the groupings do allow for perception of a clarified process if one knows the concept and strategy of the music ahead of time – and maybe even if one doesn't. But what happens in the sounding music? Is what we are hear simply a catalogue of the possibilities of a music that the list provides within this inhabitable musical space? What gives life to the piece, and what is the nature of the music?

The groupings of the n-note chords (and indeed the groupings of the internal rhythms of each collection) feature numerous interesting mathematical details. If we look carefully at the total number of chords in each subset of the octave, we can see the following sequence: there are 78 two-note chords, 286 three-note chords, 715 four-note chords, 1287 five-note chords, and so on, through the final, single thirteen-note chord. This sequence, (78, 286, 715, ...) can be found in the 13th line of the mathematical object known as Pascal's Triangle.[iii]

This relationship to Pascal's Triangle holds for each total grouping of n-note chords. The sum totals create a symmetrical structure. The number of two-note chords is the same as that of the eleven-note chords, that of the three-note chords the same as that of the ten-note chords and so on, as Pascal's Triangle describes. In contrast to this feature, it's noteworthy that musically *The Chord Catalogue* is slightly less symmetrical. There is a residue of the 13 twelve-note chords and the one thirteen-note chord at the end of the symmetrical structure, which do not have symmetrical counterparts. This is because only chords considered are those that consist of two-note chords or more, disqualifying the one 'zero-note chord' and the thirteen 'one-note chords' of the octave C4 to C5).

If we look at the groupings as Johnson has ordered them, in each sequence of n-note chords we find similar structures. These derive from the fact that Johnson divides each set of n-note chords into sub-phrases, with the top note ascending. That is, the first phrase consists only of the (unique) lowest cluster of n notes; the second phrase of all chords with the top note one one step higher, giving more option; the third phrase of all chords with the top note one step higher again, etcetera. The resulting list of two-note chords is rather simple, since the number of possibilities increases linearly with the top note, each subsequent grouping extended by one, to feature a progression from one two-note chord to twelve two-note chords: a neatly organized, logical counting that results from the process of enumeration.

But as early in as the three-note chords, other interesting things start to happen. The groupings in the three note chords reveal the triangular numbers (numbers that equal sums of numbers in a triangular arrangement, for example: of pool balls before breaking): after the lowest, three note cluster (c-c#-d), you get three possibilities with the top note at d#, six with the top note at e, ten with the top note at f, etcetera. The four-note chords give the tetrahedral numbers, which can be thought of as triangular numbers in the third dimension. Likewise, the rhythm of the groupings in the five-note sequence gives the numbers for hyper-triangles in the 4th dimension, the six-note sequence counts hyper-triangles in the 5th dimension, and so on.

While you don't really hear the math going by unless you are listening for it, what happens to the music is, in fact, a reflection of it. The polyphony intensifies as the dimensions of the geometric objects being represented increases. Sonically, the textures are crowded and intense; they grow ever more so as n-note sequences increase, leading to the final 13-note chord.

In correspondence with the author, Johnson has suggested that a composer may find the music "outside" of her or himself.[iv] Here, we can see that the list that constitutes his composition appears not only outside of the composer's subjectivity, but also in nature, with a mathematical underpinning described by a famous theorem. Johnson speaks of this, but in musical and practical terms rather than mathematical ones:

I like to think of The Chord Catalogue as a sort of natural phenomenon - something which has always been present in the ordinary musical scale, and which I simply observed, rather than invented.[v]

What can be appreciated in *The Chord Catalogue* is not simply the "list" after all, but also the formation of a lucid, formal object with many structural interests, that unfold as a highly specific play of gradually developing rhythmical patterns, which give *The Chord Catalogue* its specific musical character.

Clearly, there is much more to explore in this direction of formal composition and other composers have since begun exploring. In fact, Johnson has composed many catalogue works since *The Chord Catalogue*, but none so devoted to the idea. In *Kirkman's Ladies* (2005), he uses combinatorial designs known as "block designs" to offer a musical answer in the form of a catalogue to a mathematical problem known to the world as "Kirkman's Schoolgirl Problem" that was first posed by Rev. Thomas Penyngton Kirkman in 1850:

In a boarding school there are fifteen schoolgirls who always take their daily walks in rows of threes. How can it be arranged so that each schoolgirl walks in the same row with every other schoolgirl exactly once a week?

Another work, more recent, is *Intervals* (2013), a catalogue of harmonies that explores the all-interval tetrachord. Another, *Trio* (2005), presents all 288 three-note chords of which the notes sum to 72 when middle C is assigned the value 24. Many more of Johnson's compositional output incorporates the complete set of a particular parameter or parameters in the context of their musical content.

Johnson's pieces and other musical catalogues like it are much more than mere lists. These works go beyond the oft-seen contemporary Duchampian tendency to bring to the forefront the humble existence of a physical or conceptual object of the world and in it and place this object within the context of a deliberate aesthetic strategy to be observed and perceived as art. These works also offer new and discrete modes of perception to both the ear and the intellect. Can we say that our listening is conditioned by rationality and cognition of mathematical structures as we listen to the work? The experience might be something like looking at a Donald Judd sculpture. We are confronted with physical geometry, yet we are looking at a form of art. Could listening to musical catalogues be a kind of ontological experience as well as a simple, auditory one? What kind of proposal are we making, then?

One way to begin entering into the world of musical catalogues is to look at two works that are related to Johnson's groundbreaking catalogue in obvious ways. *Within Fourths/Within Fifths* by Samuel Vriezen and *Combinations* by the author of this article are, like *The Chord Catalogue*, each combinatoric. This pair of works also each make use of strict rhythmic logics that like *The Chord Catalogue*, are specified by local and global parameters which in themselves are a kind of catalogue, yet arrive each at very different kinds of musical results.

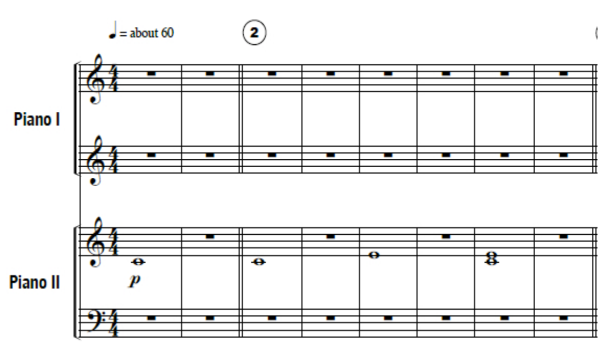
Combinations

Just like *Within Fourths/Within Fifths* and *The Chord Catalogue*, *Combinations* by the author of this article for piano four-hands addresses strict and elaborate combinatorial and rhythmic strategies. *Combinations* presents all of the 2036 combinations of the binomial coefficient, climbing from "one choose one", through "two choose one", "two choose two", "three choose one", "three choose two", "three choose three" up to and including, finally, "ten choose ten". Rhythmically, the piece integrates a nontrivial rhythmic sequence[vi] and a harmonic language that was composed without respect to a logical process.

In mathematics, combinations are described with the terminology of "n choose k". For example, when we say, "five choose three", we mean that there are a total of five notes, and that we want to choose all of the combinations of three of them. There are exactly ten possibilities:

{(1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 3, 4), (1, 3, 5), (1, 4, 5), (2, 3, 4), (2, 3, 5), (2, 4, 5), (3, 4, 5)}

The music consists of a mapping. Ten notes are selected and fixed in registration. The music begins with the first combination "one choose one" of one note (note one of ten) once - note one, C. In the next combination, "two choose one" [(1),(2)], we see notes one and two, C and E, each separated by a measure of rest, followed by "two choose two" or (1,2), consisting of notes one and two combined and played as a harmony:



This appears as a very simple mapping and as each of the ten notes are introduced and the vast number of combinations are enumerated, a much more detailed rhythmic object becomes slowly introduced, logically derived from the combinatorial formula. The combinations, as we have seen with just two notes, are broken down into subsets, separated by a measure of rest. As the combinations grow, a more sophisticated rhythmic design is established. For example, the rhythmic system throughout the work can be generalized by looking at an easy example, "5 choose 2":

This represents the following combinations, grouped in phrases with rests between them:

{(1,2),(1,3),(1,4),(1,5)}   {(2,3),(2,4),(2,5)}   {(3,4),(3,5)}   {(4,5)}

Four subsets have been formed. Each new subset contains, in order, the groups of combinations containing only note one, then only note two in the first position, then note three in the first position, and then note four in the first position. Yet the music takes on several additional characteristics. The first three groupings in the excerpt begin and proceed in half notes, but conclude on whole notes. After each grouping there is a measure of rest. These rhythmic strategies are applied strictly throughout the work. The sums of the groupings of combinations neatly count down 4,3,2,1 from the first grouping of four combinations. This logical descending counting pattern emerges from the nature of the mathematics, cementing further order on the *found object* combinations. As the patterns get longer, however, the countdown starts happening on more levels.

Following the same self-similar structure, a brief excerpt of "9 choose 5" yields the following music:

The combinations, grouped in phrases with rests between them appear like this:

{(1,2,3,4,**5**),(1,2,3,4,**6**),(1,2,3,4,**7**),(1,2,3,4,**8**),(1,2,3,4,**9**)}

{(1,2,3,5,**6**),(1,2,3,5,**7**),(1,2,3,5,**8**),(1,2,3,5,**9**)} {(1,2,3,6,**7**),(1,2,3,6,**8**),(1,2,3,6,**9**)}

The groupings of chords are still counting down neatly 5,4,3,2,1 (2 and 1 follow in the sequence and are not shown). The harmonies have become much richer and the phrases longer. Here, the subtle harmonic change that the first four (fixed) notes of the chord/combination and a moving fifth voice allow reaches deeper into the strata of combinations. Because the combinations now consist of collections that include a total of nine notes, the length and variety has altered the sonic landscape. The consistency of the rhythmic design maintains the flow of the music.

Like *The Chord Catalogue*, *Combinations* counts out the numbers of Pascal's Triangle, but here the list of combinations, given in order, counts out each of the numbers in the lines of Pascal's Triangle, all the way to the tenth line, rather than counting n-dimensional hypertriadic numbers along the Triangle's diagonals. Combinations has a different relationship to the same mathematical object, representing the fact that the Triangle consists of the binomial coefficients.[vii] This is not a trivial distinction. Where within each section, Johnson's phrases grow longer as he reads the numbers diagonally down the triangle, *Combinations* has sections that inherit the symmetry of their phrasing structure from the symmetry of each horizontal line of the triangle – though the subdivisions inside each n-note section are rhythmically organized such that the phrases become slower and more fragmented as the chords approach higher density. This leads to a very different development of energies within each phrase: ever-increasing activity and density in *The Chord Catalogue*; ever more spacious timing in *Combinations*. On the higher level of the sections themselves, though, we find the inverse relation: *The Chord Catalogue* having sections that symmetrically expand in length, then contract; *Combinations*, with its ever-increasing sections, as there are more and more notes to choose chords from. The sound that emerges in *Combinations* is very different, more spacious and much calmer; the language of the rhythmic structure is primarily what creates that distinction.

While deploying strict rules with respect to the harmonic combinations and rhythmic structure, one parameter is determined by taste: the choice of scale, which was freely composed. The ten notes chosen span four octaves and include only naturals. The emphasis appears in three areas of the keyboard, a low C major triad in 2nd inversion, a C Major Seventh chord at Middle C and two stacked fourths in the upper register. This strategy may be seen as an attempt to color the music aesthetically while allowing for the mathematics to be heard in a rational, logical, and complete way. Thus, the piece is no longer a simple list and now has one extra parameter.

Another feature that appears regularly throughout *Combinations* is the repetition of root harmonic progressions. Throughout the work this appears regularly as the music unfolds.

To take one example in Section 7, the first instance of the 3-note chords includes five harmonies:

{(1,2,3),(1,2,4),(1,2,5),(1,2,6),(1,2,7)}

In Section 9, these 3-note chords are repeated and extend two chords further, incorporating all nine pitches:

{(1,2,3),(1,2,4),(1,2,5),(1,2,6),(1,2,7),(1,2,8),(1,2,9)}

This extension and equilibrium of the harmonies enriches the logic, anchoring the familiarity of each combination and allows the development of the progressions to be detected audibly and perceived. This perception may not be the only goal and may consider an equally reasoned aesthetic of taste.

The aesthetic of taste reveals an elusive aspect of all musical catalogues. Catalogue composers must choose their global parametric constraints, and then how to present them, even within totally rationalized systems like the rhythmic grouping in *The Chord Catalogue* or, as described below, the cycle of fourths in *Logical Harmonies (1)* by Richard Glover and the rhythmic pattern of *Clapping Music* by Steve Reich. In these works, composing requires selection of registers, instrumentation, harmony, and rhythm. Every composer of a musical catalogue that chooses, for example, a logical, cyclical harmonic progression that suits the intention of the work identifies the choice that clearly could be much different, more elaborate, more inconsistent, more opaque. Are transparency and perceptibility only musical goals for seeking objectivity? What defines a less logical choice and one more so? What would be the reasons for choosing one over the other?

Within Fourths/Within Fifths

In *Within Fourths/Within Fifths*, Samuel Vriezen has constructed a catalogue work for solo piano that develops its harmonies on the basis of interval and scale structure. Taking as a basis the intervals of a perfect fourth and a perfect fifth, *Within Fourths / Within Fifths* explores, in the composer's words, "all possible chords from 1 to 5 voices, where each voice is limited to three scale positions within the interval of a fourth, with the fourths stacked.. [and] … all possible chords from 1 to 5 voices, where each voice is limited to four scale positions within the interval of a fifth, with the fifths stacked".[viii] Vriezen terms this work a "chord enumeration" piece, expressly indicating both harmonies and the idea of the "list" that Johnson expresses when he writes of *The Chord Catalogue* in his article.

The compositional strategy is effectively identical in each of the two movements. To describe how it works, it's enough to analyze the first movement, *Within Fourths*. A rudimentary example near the beginning helps us see what is happening:

The construction of the piece evolves to present, in each section throughout the piece, each chord as a sum of the n-notes in the chord. Here is how the structure unfolds: notes Bb, C, and D – the three pitches that the top voice is restricted to – are numbered 0,1, and 2 respectively. The notes of the second voice, F, G, and A, exactly a fourth below those of the top voice (hence the title of the movement), are also numbered 0,1, and 2, respectively. In this particular two-bar excerpt, the two-note chords will all equal sums of two:

A+Bb (2+0) = 2, F+D (0+2) = 2, G+C (1+1) = 2

Rhythmically, the first two chords progress from half note to whole, while the last is a whole note. Thus, the two bars both end on a whole note; in fact, all bars of the piece end on a whole note, otherwise containing half notes exclusively. The first two chords are both sums of '0' and '2', and are grouped together as a two chord phrase, while the last chord, a single sum of 1+1, forms a single unit. This ordering of entrances settles on the natural logic of "finding the music".

To discuss a more detailed example that gives us more of the music, let's move ahead to *Within Fifths* and look at Section III, a triadic section where the chord sums total four:

The first four scale positions are B5, A5, G5, and F#5 (respectively numbered 3,2,1,0). Following the same logic we discussed earlier, the descending stacked 5ths are respectively numbered (E,D,C,B = 3,2,1,0 and A,G,F,E = 3,2,1,0). The logic in the order can be easily be seen once again if we reveal the numbers – the found system (each triad from top to bottom). In the first measure:

(0+1+3),(1+0+3),(0+3+1),(3+0+1),(1+3+0),(3+1+0)

That is, the first measure gives all possibilities of adding a '0', a '1' and a '3'. Likewise, the second bar gives all possibilities with '0', '2' and '2'; the third, all possibilities with '1', '1' and '2'. This list of permutations is an example of the pattern that continues in the subsequent measures, and in fact throughout the entire piece, from start to finish.

Vriezen offers all of his progressions in half notes and whole notes, sounding harmonies for four beats only after a progression is completed. The progressions begin with single harmonies, grow longer (sometimes much longer) and then return to their single harmonies. Each section of the piece does so symmetrically until the catalogue is complete. The independent quality of each harmony within the logic of the progression is one aspect of the music that sustains interest. However, what we are really listening to is not just the length or brevity or richness or sparseness of the music, but the unfolding of a kind of map that may be considered to engage the potentialities of musical spaces with respect to determined, articulated musical qualities. Vriezen has commented on this:

I like to think of catalogues as in-time maps that completely explore spaces of musical possibilities, that themselves are parametrically defined and have a so-to-speak out-of-time existence, and the method of relating those two (in and out-of time) is what makes the life of the piece.[ix]

The rhythmic design adds much variety and temporal interest, and there are also motivic melodic structures and recurring harmonic structures that emerge. In Section III of Within Fifths, melodies that appear in the soprano voice reappear in Sections IV and V as part of slightly altered harmonies. In subsection 5 of Section III, the six-note melody F#-A-F#-B-A-B in the soprano voice comes back in subsection 6 and 8 of Section IV above identical two-note harmonies. These harmonies are sustained by a newly introduced D (a fifth lower) in subsection 8 (In subsection 6, the bass note is Bb). In subsections 10 and 11 of Section V this melody reappears amid longer progressions.

The harmonies repeat with an added G in the bass in subsection 10 and an added F in subsection 11.

There are numerous other melodic and harmonic motivic recurrences. These developments in the music create a kind of kaleidoscopic effect. The melodies that come out of the mathematics have a modal, Satie-like quality, not unlike what one might hear in the *Gymnopédies*. The mathematical logic that results in the bass notes in Section IV and Section V rewrites the earlier harmonic progressions. The voicings are identical. It's a consistent outcome that offers new hues of harmonic color within the context of logical framework. The idea and the music are married completely and equally. Does the life of the piece move closer towards the conceptual or the audible?

Phasing Catalogues

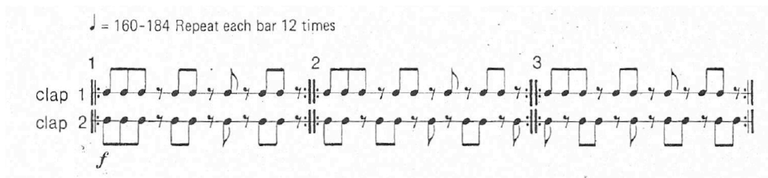
A pair of musical catalogues that take a completely different approach to cataloguing is *Logical Harmonies (1)* and *Clapping Music*, respectively by Richard Glover and Steve Reich. Here there is an attempt to separate rhythm and variation from what's perceptible and what's transformed by the compositional strategy.

Taking Richard Glover's score into consideration first, there are several notable catalogue parameters to explore. Here are the first two systems, which he notated as follows:

C	F	Bb	Eb	Ab	Db	Gb	B	E	A	D	G
C	F	Bb	Eb	Ab	Db	Gb	B	E	A	D	G
F	Bb	Eb	Ab	Db	Gb	B	E	A	D	G	C
C	F	Bb	Eb	Ab	Db	Gb	B	E	A	D	G

The symbols indicate major triads in any inversion. Scored for solo piano, the right hand plays the top line, the left hand the bottom. Glover describes the ground rules at the beginning of the score: try to stay within a two octave range, don't let your hands cross, and choose inversions that are close to each other. The first line shows both the left and right hands in rhythmic unison completing a progression of fourths, both rhythmically and harmonically. In the second system, Glover pushes the right hand backward one position; the harmonies of the two hands are again heard simultaneously. This continues throughout the piece: the process of moving the right hand one position backward, in identical rhythm, continues until the two hands are playing at unison once again. This allows the entire catalogue of harmonies resulting from this process to be heard.

This work embodies a phasing process that would be unthinkable without the concept of the canon and identifies even more closely with the early minimalist compositions of Steve Reich, essentially mimicking in structure Reich's 1972 piece for two people clapping, *Clapping Music*. In Glover, we hear harmonic movement rather than rhythmic movement. For comparison, these are the first three bars of *Clapping Music*:



This is an example of Reich's well-known phasing technique without the blurred, sparky motion of gradual phasing found in earlier pieces like *Violin Phase* and *Piano Phase*. Because each of the possible combinations of the combined patterns is used, *Clapping Music* falls under our definition of a musical catalogue. The rhythmic pattern is stated, begun in unison by two performers. Then, performer one pushes ahead to the next position while performer two continues to repeat the original pattern as in Glover's piece, where the right hand does the same. This continues until the two performers return to play the original pattern in unison once again, at which point, after twelve repetitions, the piece ends. In *Clapping Music* there are 12 beats per bar, 12 bars, and each bar is repeated 12 times. Reich's basic rhythmic pattern cycles through groups of one, two, and three beats, but the process reveals resultant patterns with complex, variable metrical figurations. In his directions to the performers, Reich notes that no accents should be made and that, "it is for this reason that a time signature of 6/4 or 12/8 is not given - to avoid metrical accents."[x] The resulting rhythmic patterns that come out of the process are what are being catalogued here. This is a kind of "found music". The composer wants to stay out of the way of the material and the process.

About the similarity of the process-oriented nature of the two works, Glover says:

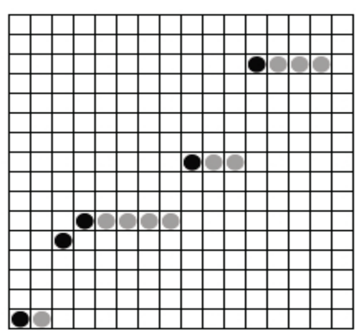
The structure is undeniably similar to Reich's phasing works, although that's not how it was conceived. I was, and am, aiming for a continuous subtle transformation in the sound; enough of a transformation for the sound to perceptibly change, but not a transformation in which the change draws attention to itself. Rather, a continuous stream of gradual change. Taking the same thing and gradually moving it out of sync with itself is an effective way of producing this kind of result, and knowing that 12-tone equal temperament has this beautiful, closed system of superimposed fifths, which can be played by two hands on the piano, gave me a perfect musical object to gradually transform. It was not designed as a 'phasing' piece per say, but that is certainly what the piece is. It provides a wonderfully 'flat' surface in terms of the performer's instructions, and allows the triads of Western tuning to do the work in generating the harmonies.[xi]

What Glover refers to as the "continuous subtle transformation" and "continuous stream of gradual change" allows us, as listeners, to encounter perceptual differences by means of ever-changing progressions. There is a kind of twisted shifting to the harmony. It's a weird spiral that has a warm, rich, natural mechanism of growth. Reich's work brings us into the realm of rhythm alone. *Clapping Music* offers discrete, repeatable segments. The music is strict, almost scientific. The listener's interest is in the kaleidoscope of pattern, repetition, and variation. Glover's idea of "gradual change" echoes Reich's idea laid out in his famous essay, "Music as a Gradual Process". Yet these are two very different pieces. The meanings of "change" (to become or make different) and "process" (a series of steps or actions taken to achieve a result) point to the contrast in strategy. Glover is after audible harmonic transformation, Reich wants systematic metrical variety.

Two Variations

The catalogues discussed so far lay out the music with all (or almost all) of the cards face up: the musical material, whether rhythmic, harmonic, melodic, and intervalic in nature, is presented in a purely logical and transparent manner. What about an approach where a musical catalogue is composed more flexibly? Paul Epstein has made a collection of pieces based on drawings by the American artist Sol LeWitt that are devoted to geometrical shapes and structures. One piece in Epstein's collection is *Drawing No.3 (Slow Title)*. This work is a catalogue, but the way Epstein presents the catalogue is selective and somewhat free.

Drawing No.3 (Slow Title), subtitled "all possible combinations of two of eight variants of a sixteen-beat pattern", is a short work for solo piano. The catalogue is inspired by wavy lines that occur in some of LeWitt's drawings, mapping pitches and durations of wavy lines onto a grid of sixteen diatonic spaces by sixteen eighth notes.[xii] Here is the grid of one of the two prime forms of a pattern he uses, with the pitch locations in black and the durations in grey, where each square equals an eighth note, giving a sixteen-beat pattern:



The generative melodic pattern, taken from the grid, is composed by locating pitches within the diatonic collection. The melodic pattern begins on C in the first square of the lower bottom left of the grid; counting vertically: C-G-A-D-Bb. The durations on the grid are greyed in, each square equaling an eighth note: the first pitch C equalling a beat, the G half a beat, the A two and a half beats, and so on.[xiii]

A look at the first phrase shows how Epstein translates his wavy line into music. From this process, he generates larger forms by rotating his grid to create variants of the mapped melody, including the inverted, retrograde, and retrograde-inverted forms of both the variant and the original, and then cataloguing the twenty-eight possible combinations of those eight forms.[xiv]

What makes the catalogue different by virtue of flexibility? In written notes to this piece, Epstein explains. He does not use a systematic order of the combinations in order to avoid immediate repetitions. He also uses each of the eight forms of his two patterns in two transpositions, each placed a fifth apart, and chooses the transpositions freely for each combination. Additionally, inverted (but not retrograde-inverted) forms of the patterns are each offset by one eighth-note rhythmically and dynamics are regularly indicated and varied.[xv] These compositional choices reflect a strong intervention by the composer. What kind of questions are posed by Epstein's flexibly composed catalogue? One might be whether or not the idea of the catalogue can be made audible. To a listener, the musical catalogue is present but opaque. Here is an example of two representative measures of his results:

As to the audibility of the catalogue and his compositional technique in works of this nature, Epstein says:

I honestly don't worry too much audibility. As much as my Drawings were influenced by LeWitt, there's a crucial and obvious difference: with the graphic works you have time to study them, find the patterns, and check to see that all the possibilities are in fact used. With the music you pretty much have to take that on faith while listening, or else go through the score with gun and camera. Analysis is certainly possible, but I'm not sure it will make much difference to the listening experience - unlike analysis of classical tonal works or even of some early minimalism.

With just about all of my recent pieces there are elements - register, dynamics, articulation, etc. - that are composed either freely or (more often) by a variety of procedural systems such as chance or serialism. The systems differ from piece to piece. They may be chosen based on rational criteria: the desire to maximize or minimize *repetition may influence the ordering of patterns and pattern combinations; or it may be just a matter of what I think will sound good.*[xvi]

Epstein's desire to avoid immediate repetition can itself be investigated systematically. Indeed, one could investigate the notion of 'maximum change' with respect to musical space. This could mean the maximum number of changes to a melody or a melodic fragment within the scope of possible, correct solutions to a counterpoint exercise, or the maximum number of chord changes in a symphonic movement taking into account some particular rule of functional harmony. Michael Winter's composition *maximum change* is an enumeration of all possibilities of a fixed four-note chord performed by up to four different instruments and there is maximum timbral change from chord to chord. More formally speaking: the maximum timbral change of a static chord of four discrete pitches as played by up to four instruments with distinct timbre, where each pitch is assigned a different instrument upon each successive event. Thus, the composition is an inquiry into timbral change.

The problem Winter has posed for himself required serious study. Mathematician Azer Akhmedov showed Winter the circumstances in which such a morphology is possible, after which Winter generated the solution and composed the work. Winter explains how the problem was solved:

In the case of maximum change, the rather simple solution used to create the piece was found independent of the proof. The solution generates a list for each of the 4 pitches that determines which instrument it is assigned over time by an algorithm that works as follows. For each pitch generate a permutation of (1,2,3,4). For the first pitch, simply repeat that (ordered) set 64 times. For the second pitch iterate the set 64 times, rotating it left by one position every iteration. For the third pitch, iterate the set 64 times, rotating it left by one position every 4 iterations. And finally, for the last pitch, iterate the set 64 times, rotating it left by one position every 16 iterations.[xvii]

Here are the first ten bars containing the first ten chords:

A few other aspects of his catalogue are worth noting. Winter defines his instruments as percussion instruments, each with long decay (such as circular plates, rectangular bars, hollow tubes, and struck strings such as crotales, glockenspiel, chimes, for example) and each being able to strike each of the four selected pitches individually and together. He also allows for transposition such that every occurrence of each pitch is transposed by the same interval such that the conglomerate chord always consists of four different pitches (in an alternate version, he allows four pre-determined pitches that are the same for the ensemble, but not those given explicitly in the first). The performers pause eight to twelve seconds between each attack, letting each tone of the chord ring freely and decay naturally. The number of chords that result from Winter's calculations total 256 and therefore the performance time is roughly forty-two minutes in length, given an average ten seconds between each attack. In the alternate version of the piece, durations between attacks are made freely and pitches are predetermined but unspecified. Unlike the other catalogues discussed, Winter allows the music to avoid being absolute with respect to the timing.

Reflecting on the audibility of the musical catalogue, Winter concludes:

I take a different stance than Tom Johnson, although I appreciate his position. Tom feels that if you can't hear the logic, then the piece has failed. I accept that someone could appreciate the piece on some other level I had not considered or on their own terms altogether. I don't gauge the success of a piece or question someone's experience based on whether or not they perceived my original concept. Regardless, like Tom, the logic of my pieces are never divorced from a musical investigation.[xviii]

Approaching Completeness

There is an extraordinary variety to the approaches and musical results of these catalogues. In *The Chord Catalogue*, the idea of the "list" and the "object" appears. This touchstone work from 1985 offered a fundamental seed into considerable expansion of the catalogue idea. In a simple yet visionary work there were hints of great complexity and the latent potential of similar musical spaces. Another smart combinatorial project, *Within Fourths/Within Fifths*, helped us see how musical catalogues can be thought of as maps that explore musical spaces, and how this insight provided a deeper assessment of the catalogue project. *Combinations* shows that a catalogue with strict combinatorial and rhythmic logic can take yet even another shape, and introduced the idea that subjectivity manifests in the choosing of a scale according to taste, plays a role in all musical catalogues. *Logical Harmonies* and *Clapping Music* showed how two works can be constructed by utilizing an identical process, and still have a very different compositional goal and very different musical results. *Drawing No. 3* showed how a musical catalogue can be constructed flexibly and without transparency. *maximum change* researched how timbral change can be conceived of as a strict parameter, further bringing out the variety and intelligent, rigorous design evident in each of these catalogues.

Studying this meta-list of musical catalogues, it's easy to conclude that there is much less consistency and rigor in such a meta-catalogue than there is in the individual catalogue works themselves. What draws the musical catalogues together is the principle of exploring every possibility of at least one parametric feature of a musical object, but every possibility could never be accounted for in the meta-list.[xix] In light of this unusual characteristic of the meta-list of musical catalogues, how can these catalogues we've discussed be considered?

The attempt to compose a musical catalogue puts trust in two fundamental ideas: that the entire set of compositional possibilities is somehow more interesting or fulfilling to explore than some subset of the possibilities, and that exploring the entire set solves a condition imposed by the parametric space of the musical objects presented. Tom Johnson offers one explanation:

I often like to try to do all the possibilities of something, so that there is a reason to stop the music somewhere, and I always feel more sure of myself if some mathematician confirms that I have done all of the possibilities.[xx]

That explanation, while addressing the logic of the catalogue project, partially relies on another branch of knowledge, mathematics. This reliance doesn't explain the utility of the condition imposed, though. It points to a deeper interest in completeness with respect to mathematics and the characteristics of logic, rationality, and order. But composing music with an interest in mathematics and logic doesn't have to include every possibility of anything and indeed there are great numbers of works in which composers explore mathematical, logical spaces without composing a complete catalogue.

All of these catalogues, including Johnson's own *Chord Catalogue*, go beyond mathematical, logical features, leading into considerations of poetics and psychology.Composers of musical catalogues make complete and closed universes out of a single atom - the idea and the behavior of executing the idea. But it would probably be hard to tell to what degree this psychology informed the work without deeper analysis; this will have to be left to explore in future articles.

One question worth considering more deeply is that of perception. Can composers convey the central idea of the catalogue to the listener as a sounding music? Can the listener perceive the completeness of a musical catalogue simply by listening? In some catalogues, like *Drawing No.3*, one would almost definitively have to know how the music was constructed first. In a piece like *Clapping Music*, it's probably not easily apparent to most that the music gives each possible eighth-note variation of two identical parts without knowing this feature ahead of time. Yet in works of total transparency like *The Chord Catalogue*, *Combinations*, and *Within Fourths/Within Fifths* there is no guarantee that the concept will be heard without foreknowledge, either. Is knowing that one is hearing a musical catalogue while listening to a given piece of music important to reception of the music? Both Epstein and Winter, for example, state explicitly that from their point of view as composer this does not figure into things whereas Johnson and Glover may believe that this kind of perceived logic is what's most relevant.[xxi]

These questions posed are just a few to be answered; there are many more to be asked. The existence of these questions and these works show that the new tradition of musical catalogues has life to it. This new tradition will very probably continue to bear much good music and stimulating discussion far into the future, and that is an immensely exciting prospect.

[i] Tom Johnson, "I Want to Find the Music", essay, undated. See www.editions75.com
[ii] Tom Johnson, program notes to New Music America Festival in Houston, 1985
[iii] Pascal's Triangle is a number triangle where numbers are arranged in rows as the result of a particular mathematical formula studied by Blaise Pascal. It had been described about 500 years earlier by Chinese mathematician Yanghui, as well as by the Persian astronomer-poet Omar Khayyám, It is therefore known as the Yanghui triangle in China. For a detailed description of this formula, see: http://mathworld.wolfram.com/PascalsTriangle.html
[iv] Tom Johnson, email communication to the author, July 18, 2009
[v] Tom Johnson, liner notes to CD: "The Chord Catalogue – Within Fourths/Within Fifths," Edition Wandelweiser r recording, EWR 1304
[vi] A "non-trivial" sequence refers to the mathematical concept of triviality, where a solution to a system is too obvious to be of intellectual value. Michael Winter, a composer and mathematician, described the rhythmic sequence in this work as non-trivial upon programming it.
[vii] Also see: Weisstein, Eric W. "Binomial Coefficient.": http://mathworld.wolfram.com/BinomialCoefficient.html
[viii] Samuel Vriezen, *Within Fourths/Within Fifths*, musical score, 2006. See http://sqv.home.xs4all.nl
[ix] Samuel Vriezen, email communication to the author, January 7, 2014
[x] Steve Reich, *Clapping Music.* Vienna: Universal Edition, 1972.
[xi] Richard Glover, email communication to the author Jan. 7, 2014
[xii] When the dots on the grid are connected with drawn curves, the wavy line appears. This is the wavy line that is transformed from LeWitt's drawings. These are hand- drawn lines that were purposefully irregular, but with strictly defined end-points (LeWitt created a mix of the handmade and the mechanical).
[xiii] The grid could also have consisted of chromatic steps, uneven durations, and more. The parametric choices are selective, but are also constrained by the limitations of the grid and the availability of possible melodic transformations.
[xiv] Here we can refer again to the binomial coefficient: "eight choose two" - that is, all of the combinations of eight melodies in pairs of two at a time.
[xv] Paul Epstein, "*Drawings* (after Sol LeWitt): A Neo-Minimalist Tribute", musical score, self-published, 2011
[xvi] Paul Epstein, email communication with the author, October 11, 2014
[xvii] Azer Akmedov and Michael Winter, "Chordal and Timbral Morphologies using Hamiltonian Cycles," *Journal of Mathematics and Music*, Vol 8, Issue 1 (2014): 1-24
[xviii] Michael Winter, email communication to the author, October 28, 2014
[xix] The feature of disparateness in meta-catalogues is not unusual and has been discussed elsewhere in another artistic realm, that of concrete poetry, by German poet Ann Cotten.
[xx] Tom Johnson, *Looking At Numbers (*Springer, New York, 2014). Chapter 9, "Loops", p.94
[xxi] At the same time, the titling of *maximum change* and Drawing No. 3 (all possible combinations of two of eight variants of a sixteen-beat pattern) immediately tell us something essential about their musical catalogues.

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